Interactive verification of Markov chains: Two distributed protocol case studies

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Introduction

- In the interactive theorem prover

Isabelle

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- Verified two case studies

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  - ZeroConf protocol (IPv4 address allocation)

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- Verified two case studies
  - ZeroConf protocol (IPv4 address allocation)
  - Crowds protocol (anonymizing service)
- Built on Isabelle's probability theory and Markov chains
  Hölzl & Heller (ITP 2011), Hölzl & Nipkow (TACAS 2012)

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- Proof language and proof methods

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- Powerful proof methods
  (rewrite engine, Sledgehammer, ...)
- Important theories: datatypes, real analysis, measure theory, probability theory, Markov chains, ...

Case study: ZeroConf protocol

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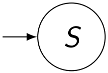
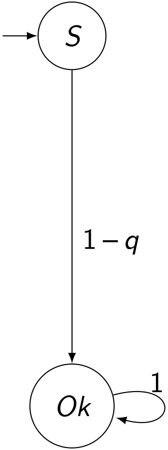
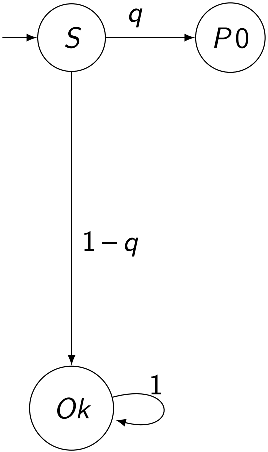
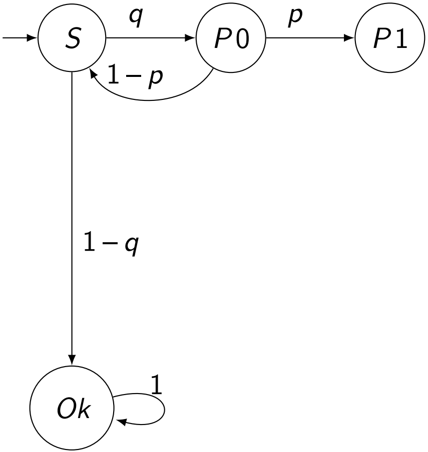
- Model checking analysis of Kwiatkowska *et al.* (2006) and Andova *et al.* (2003)

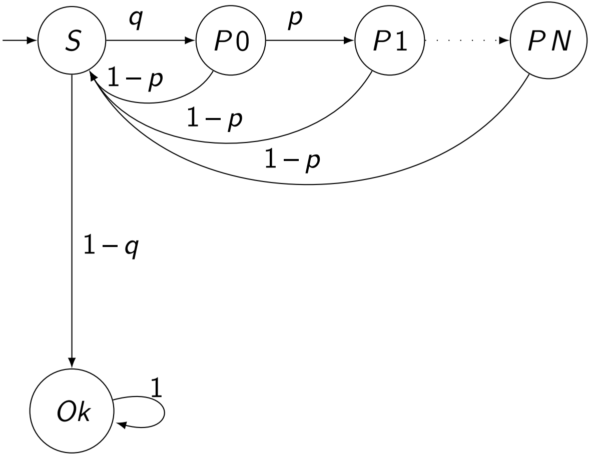

S


S
1 − q
Ok
1

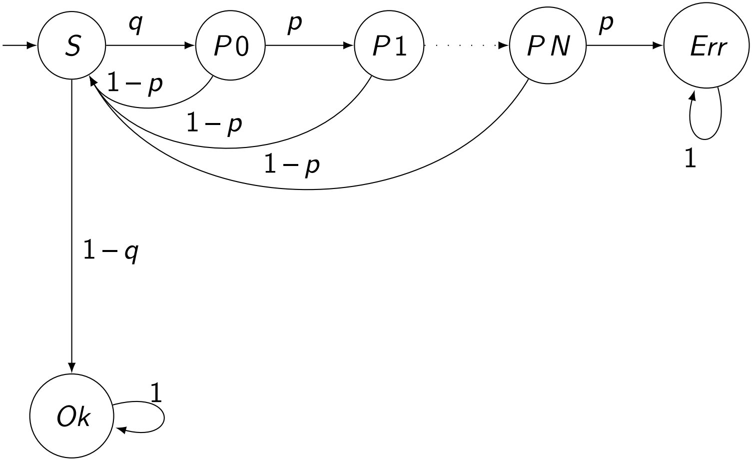


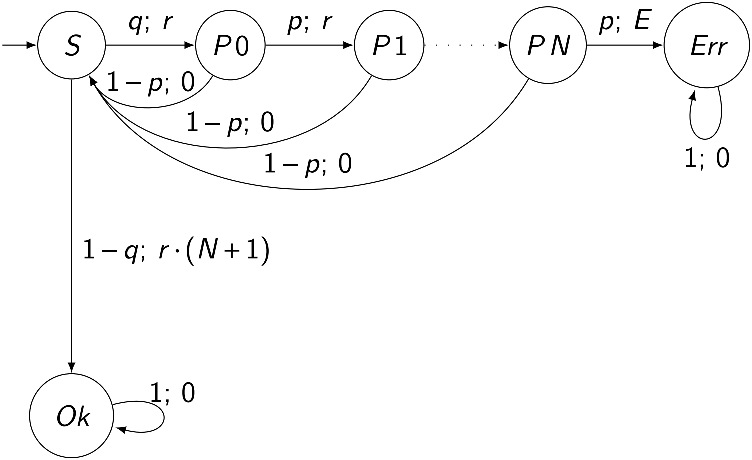
q
S
$P0$
$1-q$
Ok
1


S
P0
P1
Ok
q
p
1 − p
1 − q
1



S
q
P0
p
P1
PN
1 − p
1 − p
1 − p
1 − q
Ok
1


S
q
P 0
p
P 1
P N
p
Err
1 − p
1 − p
1 − p
1
1 − q
Ok
1


S
q; r
P0
p; r
P1
PN
p; E
Err
1 − p; 0
1 − p; 0
1 − p; 0
1; 0
1 − q; r · (N + 1)
Ok
1; 0

- Fix parameters:

  fixes $N :: \mathbb{N}$ **and** $p\ q\ r\ E :: \mathbb{R}$

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- Define state space:

$$\textbf{datatype}\ \textit{zc-state} = S \mid P\ \mathbb{N} \mid Ok \mid Err$$

$$\Omega = \left\{S, Ok, Err\right\} \cup \left\{P\,n \;\middle|\; n \le N\right\}$$

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- Define the transition function τ:

$$\begin{array}{llcl} \tau\ S & Ok & = & 1 - q \\ \tau\ S & (P\ 0) & = & q \\ \tau\ (P\,n) & (P\,(n+1)) & = & \text{if } n < N \text{ then } p \text{ else } 0 \\ & & \vdots & \end{array}$$

- Defines a Markov chain:

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lemma *markov-chain* $\Omega\ \tau$

- Probability theory gives us:

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$$\mathrm{P_{err}}\ s = \mathrm{Pr}_s(\omega.\ \exists n.\ \omega\ n = \mathit{Err})$$

- Analyse: $\mathrm{P_{err}}\ S = ?$

lemma

$$n \le N \implies \mathsf{P}_{\mathsf{err}}\ (P\,(N-n)) = p^{n+1} + (1-p^{n+1}) \cdot \mathsf{P}_{\mathsf{err}}\ S$$

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case $(n+1)$

have $\mathsf{P}_{\mathsf{err}}\ (P\,(N-(n+1)))$

$$= p \cdot (p^{n+1} + (1-p^{n+1}) \cdot \mathsf{P}_{\mathsf{err}}\ S) + (1-p) \cdot \mathsf{P}_{\mathsf{err}}\ S$$

by (*simp*$\cdots$)

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also have $\ldots = p^{(n+1)+1} + (1 - p^{(n+1)+1}) \cdot \mathrm{P_{err}}\ S$

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qed

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- 16 hosts ($q = 16/65024$), 3 probe runs ($N = 2$), $p = 0.01$:

corollary $$\mathrm{P_{err}}\ S \leq 10^{-13}$$

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- Similar to τ define the cost function ρ:

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- 16 hosts, 3 probe runs, $p = 0.01$, $r = 2ms$, $E = 3600s$:

theorem $\mathrm{C_{fin}}\ S \leq 0.007$

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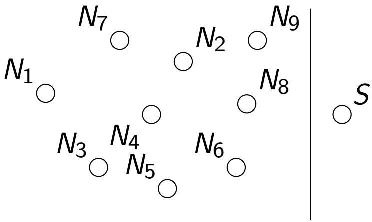
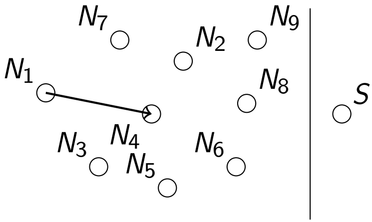
  - Information a contacted collaborating node gains is small


N_7
N_9
N_2
N_1
N_8
S
N_4
N_3
N_6
N_5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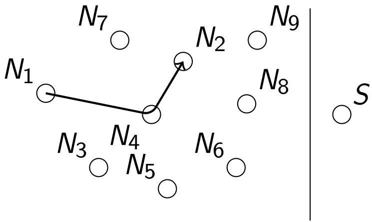

N_7
N_9
N_2
N_1
N_8
S
N_4
N_3
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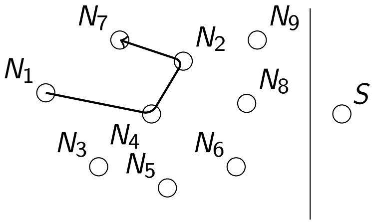
N_5


N_7
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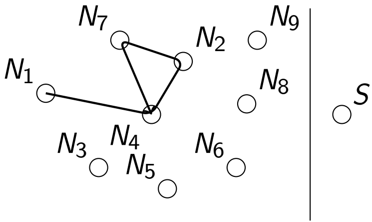
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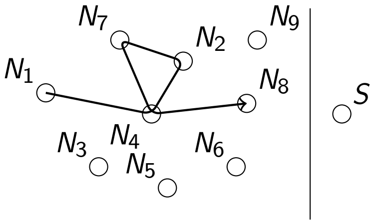
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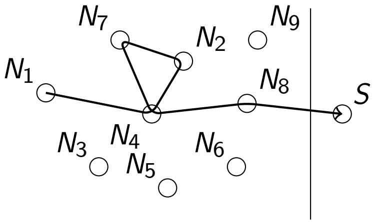
N_7
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N_1
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N_3
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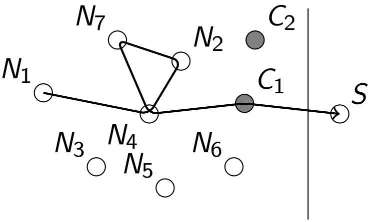
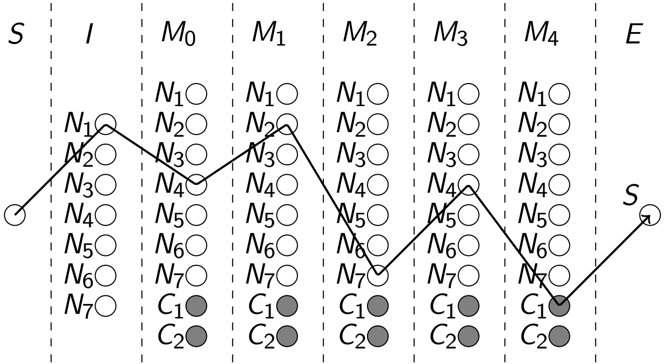
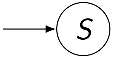
N_6



N_7
C_2
N_2
N_1
C_1
S
N_4
N_3
N_6
N_5


S
I
M_0
M_1
M_2
M_3
M_4
E
N_1 N_2 N_3 N_4 N_5 N_6 N_7
C_1 C_2
S

Probabilities:



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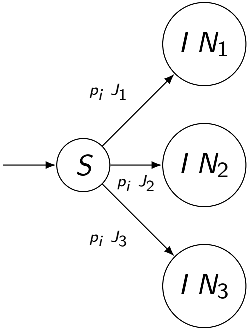
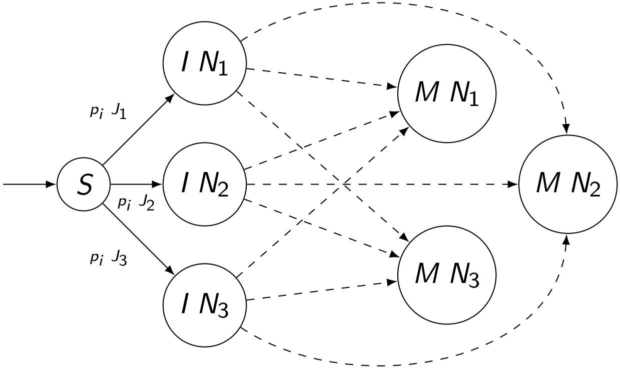
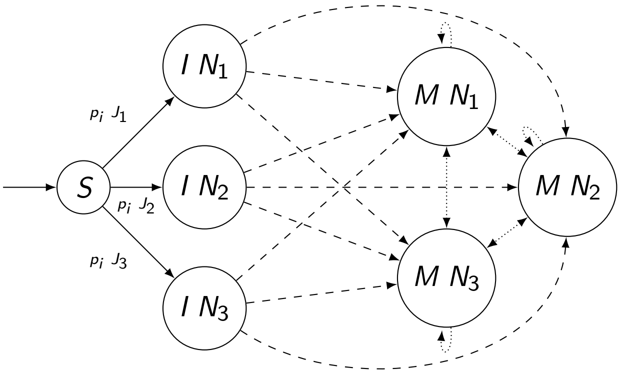


Probabilities:

1/3





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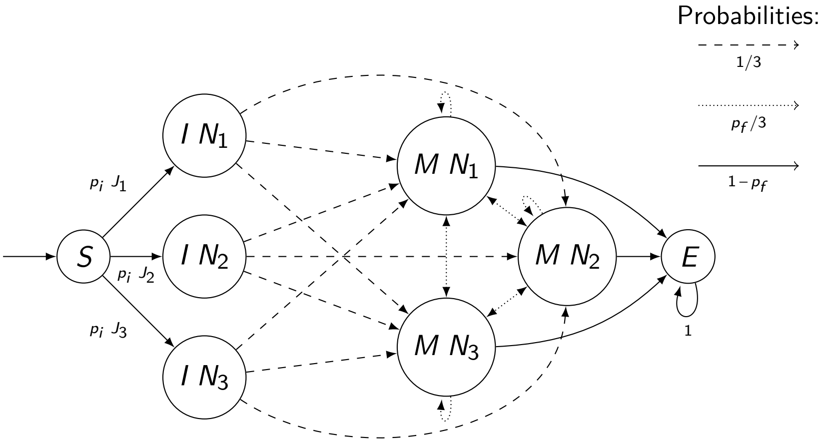
$p_f/3$


Probabilities:
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$M\ N_1$
$M\ N_2$
$M\ N_3$
E
$p_i\ J_1$
$p_i\ J_2$
$p_i\ J_3$
1

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$$\textbf{fixes}\ N\ C :: \textit{node set}\ \textbf{and}\ p_f :: \mathbb{R}\ \textbf{and}\ p_i :: \textit{node} \rightarrow \mathbb{R}$$

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- Define state space

$$\textbf{datatype } \alpha\ \textit{c-state} = S \mid I\ \alpha \mid M\ \alpha \mid E$$

$$\Omega = \left\{S\right\} \cup \left\{I\ n \,\middle|\, n \in N \setminus C\right\} \cup \left\{M\ n \,\middle|\, n \in N\right\} \cup \left\{E\right\}$$

- Define transition function

$$
\begin{aligned}
\tau\ S &\quad (I\ n) &&= p_i\ n \\
\tau\ (I\ n) &\quad (M\ n') &&= 1/|N| \\
\tau\ (M\ n) &\quad (M\ n') &&= p_f/|N| \\
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- Prove Markov chain property

theorem *markov-chain* $\Omega\ \tau$

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- Information the collaborating nodes gain when contacted

theorem $I_{\mathit{hit}}(\mathit{init}; \mathit{last\text{-}ncoll}) \leq \left(1 - \frac{|N \setminus C| - 1}{|N|} \cdot p_f\right) \cdot \log_2 |N \setminus C|$

Related Work: probability theory in ITPs

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  Coble (2009)

- Formalization of pGCL (prob. & non-det. language)
  Hurd *et al.* (2005), Audebaud & Paulin-Mohring (2009)

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Slides available at: `http://www.in.tum.de/~hoelzl`